Non-separable Boundary Degrees of Freedom as Physical Beables in a Scalar–Gauge Field Theory

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A realist, ontology-first formulation of a scalar–gauge field theory is developed in which physically meaningful degrees of freedom are not associated with pointlike variables or region-factorisable subsystems, but with an extended boundary configuration. In this approach, a domain wall Σ, generated by a global interpolation of the scalar field, together with an anomaly-fixed odd integer sector label L, determined by the gauge field, jointly form a single non-separable beable in the sense of Bell. Neither Σ nor L is sufficient for ontic status in isolation; only the coupled scalar–gauge structure (Σ, L) possesses the completeness and persistence required of a beable.

Once nucleated, this scalar–gauge boundary exhibits structural stability: its existence and sector label cannot be created, removed, or altered by any compactly supported perturbation of either the scalar or the gauge field. Boundary excitations, fractionalised charges, chiral surface modes, and other fluctuation-generated degrees of freedom depend entirely on (Σ, L) and therefore lack the autonomy necessary to qualify as fundamental ontic elements.

Within this framework, the failure of Bell-factorisation is accounted for without departing from microcausality. The extended beable spans the spatial regions whose measurement outcomes would otherwise be required to factorise, and the resulting non-separability is structural rather than dynamical. No form of superluminal signalling, hidden influence, or violation of relativistic locality is introduced. The present work is confined to ontological considerations and does not rely on Chern–Simons actions, monotonicity formulas, or mass-gap arguments. Its purpose is to isolate the minimal ontic content of the scalar–gauge theory and to show how a single global beable (Σ, L) satisfies Bell's criterion for "beables" while dissolving the separability assumption that underlies Bell-locality.

I. INTRODUCTION: ONTOLOGY BEFORE DYNAMICS

The interpretation of quantum theory remains unsettled because its empirical predictions can be accommodated by several mutually incompatible ontologies. Bell argued that any adequate physical theory should specify beables—entities that exist independently of measurement and that provide the basis for the theory's probabilistic structure [1]. Standard quantum mechanics does not explicitly furnish such entities, and the main realist alternatives—Bohmian mechanics, many-worlds interpretations, and spontaneous-collapse models—each rely on auxiliary assumptions or introduce additional degrees of freedom whose empirical status remains uncertain.

A different route is taken in the present work. Rather than beginning with quantum mechanics, attention is directed to a relativistic scalar–gauge field theory whose vacuum admits extended, topologically constrained boundary configurations. These boundaries, or domain walls, arise when the scalar field interpolates between distinct bulk sectors, while the accompanying integer-valued label L is fixed by anomaly inflow associated with the gauge field at the interface. It is shown that the combined scalar–gauge structure satisfies Bell's criteria for beables without invoking the assumptions that give rise to Bell's theorem. Because the domain wall is spatially extended and its sector integer is a global topological invariant, the configuration does not admit decomposition into subsystem components associated with spatially disjoint regions. The resulting ontology is non-separable in a precise field-theoretic sense.

The scope of the present work is restricted to establishing the ontic content of the scalar–gauge theory. The extended boundary Σ, generated by a global topology-changing interpolation of the scalar field, together with its odd integer sector label L, fixed by anomaly inflow of the gauge field, is identified as the unique non-separable beable of the theory. This scalar–gauge structure is distinguished from ordinary field excitations: it is nucleated only through a global restructuring of the scalar configuration and cannot be altered or destroyed by any compactly supported perturbation of either field. Localised modes residing on Σ—including fractionalised charges, chiral surface currents, and other fluctuation-generated excitations—depend entirely on the existence of (Σ, L) and therefore lack the autonomy or persistence required for ontic status.

By isolating the minimal ontology of the theory prior to the introduction of any dynamical considerations, a methodological stance is adopted that is analogous to the initial step in Perelman's work on Ricci flow, in which the underlying geometric structure is stabilised before any monotonicity formulas or surgical operations are introduced [2]. In the present context, clarification of the ontic content precedes any discussion of measurement, nonlocal correlations, or mass-gap behaviour. The analysis is therefore restricted to identifying the entity that carries physical reality in the scalar–gauge model. Issues of dynamics, measurement processes, or temporal evolution are not considered here; only the conceptual and structural groundwork required for a coherent ontology is established.

II. THE SCALAR–GAUGE BOUNDARY AS A PHYSICAL STRUCTURE

A relativistic field theory is considered in which a scalar field Φ is coupled to a non-Abelian gauge field A_μ on a four-dimensional spacetime manifold M. The scalar potential admits at least two distinct bulk configurations, or vacua, between which Φ may interpolate. When the scalar field crosses a critical surface in configuration space, a global interpolation is formed that generates an extended interface separating regions in different bulk sectors. This hypersurface is referred to as the scalar–gauge boundary or domain wall Σ. Its existence is a structural feature arising from the global configuration of the scalar field and is not to be interpreted as a dynamical excitation.

The boundary Σ is associated with a mismatch between the topological data of the bulk regions that it separates. This mismatch is encoded by an odd integer L that labels the sector of the boundary and is fixed by anomaly-inflow constraints arising from the gauge field at the interface [3–5]. The integer does not originate from any local property of Σ; instead, it reflects a global compatibility condition between the gauge configurations in the adjoining bulk regions. Once the interpolation of the scalar field has been established, the coupled structure (Σ, L) forms an extended geometric–topological configuration whose identity is maintained throughout the entire period in which the interpolation persists.

It is important to distinguish the hypersurface Σ from standard solitonic solutions or defect cores. The domain wall does not constitute a localised concentration of energy or a particle-like excitation; rather, it is an extended interface across which the scalar field transitions between distinct bulk regimes. Its existence is determined by the global behaviour of Φ, and its stability does not arise from an energetic minimum but from the impossibility of eliminating the interpolation through any compactly supported deformation of the fields. A local perturbation may alter the geometry of Σ in its vicinity, but it cannot remove the interface or modify the integer L that characterises its sector.

The domain wall thus serves a role analogous to that of a topologically protected boundary in condensed-matter systems, such as the interfaces between distinct quantum Hall phases or the boundary modes of topological insulators [6, 7]. In such systems, the boundary inherits global structure from the adjoining bulk regions and cannot be understood as an excitation defined solely within a local neighbourhood. The scalar–gauge boundary exhibits a similar dependence on global bulk data. However, in contrast with the condensed-matter case, the boundary in the present framework is not regarded as an emergent subsystem of an underlying material medium; rather, it is identified as the ontic element of the field theory itself.

In summary, the scalar–gauge boundary Σ arises from a global interpolation of the scalar field between distinct bulk sectors, while its odd integer label L is fixed by anomaly inflow associated with the gauge field. The coupled structure (Σ, L) thereby constitutes a single extended configuration whose identity is global, non-separable, and insensitive to any compactly supported perturbation of the fields. This scalar–gauge boundary will be shown in the following section to provide the natural candidate for a beable in Bell's sense.

III. THE INTEGER SECTOR L AS THE PHYSICAL SECTOR LABEL

The boundary Σ defined in the previous section is not specified solely by its geometric embedding in spacetime. It also carries a discrete label $L \in \mathbb{Z}$ that reflects a global constraint linking the bulk regions it separates. This integer sector is not a local observable and is not associated with any degree of freedom residing solely on Σ. Instead, it encodes a global compatibility condition between the gauge configurations on either side of the boundary and is fixed by the anomaly-inflow structure arising from the scalar–gauge coupling [3–5].

The requirement that the combined bulk–boundary system be anomaly-free imposes the constraint that L must take an odd integer value. This parity condition is familiar from topological phases of matter, in which boundary degrees of freedom appear in combinations that cancel parity anomalies present in the bulk theory [6, 7]. In the present context, the condition plays an analogous structural role: it guarantees the consistency of the scalar–gauge configuration across the interface but does not imply the existence of dynamical boundary modes. The integer L is therefore a global structural invariant of the field configuration rather than an emergent excitation.

The value of L is fixed when the scalar field undergoes the global transition that nucleates the boundary. Once assigned, this integer cannot be altered by any compactly supported perturbation of the scalar or gauge fields. Deforming the geometry of Σ, introducing localised excitations on the interface, or modifying the gauge configuration within a bounded region all leave L unchanged. A change in sector would require a global reconfiguration of the scalar field that eliminates or replaces the boundary as a whole. The identity of the boundary is therefore encoded jointly by the geometric hypersurface Σ and its sector integer L, and these two components cannot be separated.

It is important to emphasise that L does not function as a hidden variable or as a latent dynamical degree of freedom. It obeys no evolution equation and carries no probabilistic weight in the sense of quantum measurement theory. Instead, L serves as a sector identifier, specifying which global branch of the scalar–gauge configuration is occupied during the existence of the boundary. In this respect, it is analogous to topological invariants in condensed-matter systems, such as Chern numbers associated with bulk bands, which label distinct phases rather than describing local excitations [6].

Because L is fixed globally and cannot be decomposed into contributions associated with disjoint spatial regions, the combined boundary (Σ, L) is intrinsically non-factorisable. No pair of spatially separated observers can assign independent sector values to different portions of the interface or to hypothetical subsystems within it. The integer label binds the entire hypersurface into a single ontic object with a unified identity. This structural non-separability will become relevant when Bell's locality conditions are examined in Sec. VI.

In summary, the integer sector L is a global invariant fixed by the anomaly-inflow constraint and serves as the structural identity label of the boundary. It is not a local observable, not an excitation, and not a hidden variable. Together with the hypersurface Σ, it forms the coupled structure (Σ, L) that defines the non-separable ontic beable of the scalar–gauge model.

IV. WHY BOUNDARY EXCITATIONS AND FRACTIONALISED CHARGES ARE NOT BEABLES

The extended boundary Σ, together with its anomaly-fixed integer sector L, forms a single non-separable structure determined by global features of the scalar–gauge configuration. Additional degrees of freedom that may arise on or near the boundary—such as fractionalised charges, chiral surface modes, or fluctuation-driven excitations—do not possess this ontic status. In what follows, the key distinctions are identified that prevent such excitations from serving as beables in the sense required by Bell.

First, boundary excitations lack autonomy. Their existence is entirely contingent upon the presence of the boundary Σ. When the scalar field no longer interpolates between distinct bulk sectors and the boundary is consequently removed, such excitations disappear together with it. This dependence contrasts with the structural robustness of the coupled configuration (Σ, L), whose identity is preserved for the full duration of the interpolation and cannot be eliminated or altered by any compactly supported perturbation of the fields.

Second, boundary excitations do not possess global identity conditions. Fractionalised or chiral modes on Σ can be created, displaced, or removed through local fluctuations of the fields restricted to the interface. Their multiplicity and configuration are determined by local energetic and geometric features rather than by global topological data. As a result, such modes cannot support an ontic decomposition of the system into spatially distinct components: their presence at any location on Σ does not impose structural constraints on distant regions of the boundary.

Third, boundary excitations do not encode the anomaly-inflow constraint. The odd integer sector label L arises from a global compatibility condition between the gauge configurations on either side of Σ and is therefore insensitive to the local behaviour of fractionalised or chiral modes. The value of L cannot be inferred from, or reduced to, the dynamics of excitations residing on the boundary. Although such modes may signal the presence of the interface, they do not determine its sector and thus cannot constitute, individually or collectively, the ontic identity of the boundary.

Fourth, boundary excitations are not suitable candidates for beables because they admit subsystem factorisation. Degrees of freedom associated with spatially separated portions of Σ can always be specified by assigning which excitations reside within each region. This partitionability is incompatible with the non-separable character required of a beable in the present model. In contrast, the scalar–gauge boundary cannot be decomposed into components that carry independent identities or that permit distinct ontic descriptions.

Finally, boundary excitations do not satisfy Bell's requirement that beables be entities that "exist" independently of measurement [1]. Boundary modes can be created or removed through local operations and depend sensitively on perturbations of the fields, so their presence is not guaranteed in all physically admissible configurations. They therefore lack the persistent identity required to function as carriers of physical reality in the foundational sense relevant to the present framework.

It follows that boundary excitations—whether fractionalised charges, chiral surface modes, or other fluctuation-induced degrees of freedom—do not qualify as beables. Their dependence on the existence of the boundary, the absence of any global identity condition, and their susceptibility to local perturbations exclude them from serving as fundamental ontic constituents of the theory. The only viable candidate is the extended, globally identified boundary (Σ, L) itself, which will now be examined in order to clarify its non-separable structure.

V. NON-SEPARABILITY OF THE BOUNDARY BEABLE

The defining feature of the pair (Σ, L) is its non-separability. In contrast to ordinary field configurations that admit a decomposition into contributions supported on spatially disjoint regions, the scalar–gauge boundary possesses no such factorisation. The integer sector L is a global invariant that arises from a bulk-to-bulk compatibility constraint and cannot be expressed as a sum, coarse-graining, or local aggregation of quantities defined on subsets of Σ. Likewise, the hypersurface Σ is created as a single connected interface generated by a global interpolation of the scalar field, and its identity persists across its full extent for the duration of that interpolation. Because neither Σ nor L can be partitioned into components associated with spatial regions, the coupled structure admits no decomposition into spatial subsystems and therefore constitutes a single, non-factorisable beable.

This non-separability has consequences for both the ontology and the operational analysis of the theory. Observers situated in spacelike separated regions cannot assign distinct "local values" to putative portions of the boundary, as such assignments would conflict with the global constraints that define (Σ, L). Any attempt to regard disjoint segments of the interface as independent carriers of information, or as separate ontic components, is incompatible with the scalar–gauge configuration. The identity of the beable is therefore intrinsically holistic.

It is equally important to distinguish this non-separability from nonlocal interaction. Nothing in the definition of (Σ, L) permits instantaneous influence or signalling. The non-separability arises instead from structural unity: the global configuration of the scalar field fixes the existence and continuity of the hypersurface Σ, while the gauge field supplies the anomaly-inflow constraint that determines its odd integer sector L. No local operation can alter either component. Local deformations may change the geometry of Σ within a bounded region, but the continued existence of the interface and the value of its integer sector cannot be modified without a global reconfiguration of the scalar field that eliminates the boundary entirely. The non-separability is therefore topological and structural in origin, not dynamical.

This distinction also clarifies why perturbation-driven excitations do not contribute to the definition of the beable. Localised disturbances of the scalar or gauge field may generate transient boundary modes, fractionalised charges, or chiral currents, but such modes are parasitic on the existence of Σ. They do not determine its identity and disappear when the boundary collapses. A compactly supported perturbation can create or reshape these excitations, yet it cannot create, remove, or modify the global beable itself. The nucleation of Σ requires a transition of the scalar field across a global critical surface in configuration space, and once nucleated, the coupled structure (Σ, L) is insensitive to bounded perturbations. This asymmetry—local operations can produce excitations but cannot affect the ontic identity of the boundary—is essential for distinguishing the beable from the degrees of freedom it supports.

Since (Σ, L) cannot be decomposed into independent parts, it is the only structure in the theory that satisfies Bell's requirement for an ontic entity capable of grounding definite outcomes without invoking hidden variables or subsystem independence [1]. Field excitations fail this criterion because they can be created, modified, or annihilated through local operations and therefore lack persistence. In contrast, the boundary beable exists for the full duration of the interpolating configuration and cannot be removed without a global reconfiguration of the scalar field that restores a single bulk sector across the region. Its identity is thus stable under local operations and remains unaffected by measurement interactions.

The scalar–gauge boundary is therefore non-separable not because it transmits influences across space but because it is defined only as a unified structure. Its existence depends on the global configuration of the scalar field, and its identity is fixed by an integer sector that admits no decomposition into regional contributions. This structural unity is what allows (Σ, L) to function as the beable of the theory and is the feature that makes it possible to understand how Bell's locality assumptions fail without appealing to nonlocal dynamics.

VI. HOW THE NON-SEPARABLE BEABLE AVOIDS BELL'S LOCALITY CONSTRAINTS

Bell's theorem demonstrates that any theory reproducing the correlations observed in quantum spin–entanglement experiments cannot satisfy, simultaneously, the assumptions of local causality and statistical factorisation for spacelike separated regions [1]. This result is commonly interpreted as requiring nonlocal influences, hidden variables, or an Everettian branching structure to account for quantum correlations. In the scalar–gauge model developed here, none of these conclusions is necessary. The failure of Bell's factorisation assumption arises instead from the ontology of the theory: the beable (Σ, L) is intrinsically non-separable and therefore cannot support the independence conditions on which Bell's argument relies.

Bell's factorisation assumption requires that, conditioned on a complete specification of the underlying reality λ, the joint probabilities for outcomes recorded in two spacelike separated regions R_A and R_B satisfy

$$\begin{aligned} &P\big(a(R_A), b(R_B) \,\big|\, A(R_A), B(R_B), \lambda\big) \\ &\quad = P\big(a(R_A) \,\big|\, A(R_A), \lambda\big)\, P\big(b(R_B) \,\big|\, B(R_B), \lambda\big). \end{aligned} \tag{1}$$

This expression presupposes that λ can be treated as a common specification of the underlying reality that supports region-associated ontic components—often written schematically as $\lambda = (\lambda_{R_A}, \lambda_{R_B})$—so that the outcome statistics in each region depend only on the local setting and the local part of the underlying reality. In the scalar–gauge model, such a decomposition is not available: the complete specification of the underlying reality is the single global entity (Σ, L), which cannot be factorised into contributions associated with R_A and R_B. The failure of factorizability therefore arises from the ontic unity of the beable rather than from any nonlocal influence.

The failure of factorizability arises not from any nonlocal influence but from the absence of independent region-subsystems. The beable does not transmit information between separated regions, since it is not an entity that carries messages or interacts causally across space. Rather, it is a single extended structure whose identity spans the entire interface Σ. Measurements performed in distinct regions therefore probe different aspects of the same ontic object, and the resulting correlations reflect this shared origin. In this respect, the scalar–gauge model is realist yet non-separable.

This structure leaves local causality intact. The fields at spacelike-separated points obey the same local equations of motion that govern any relativistic field theory, and no superluminal signals or retrocausal influences are introduced. Microcausality remains unaltered. The non-separability of the beable affects only the statistical structure of joint outcomes, not the causal properties of the underlying dynamics. In this respect, the model rejects the assumption of "local beables" in Bell's sense without modifying the locality conditions that regulate relativistic field evolution.

Because the identity of (Σ, L) is fixed globally and cannot be altered by bounded perturbations, the correlations observed in spacelike separated measurements reflect structural unity rather than dynamical transmission. This is analogous, in a broad sense, to quantum entanglement in algebraic quantum field theory, where correlations arise because the global state is not a tensor product of states associated with spacelike separated local algebras [8, 9]. The distinction in the present framework is ontological rather than algebraic: the non-factorisability resides not in a non-product state but in the global nature of the beable itself.

The scalar–gauge model therefore evades Bell's constraints in a minimal way. It supplies a realist ontology that lacks the separability structure required by Bell's factorisation assumption while preserving the standard causal restrictions of relativistic field theory. No new interactions are introduced, no hidden variables are postulated, and no Everettian branching is invoked. The non-separable beable (Σ, L) provides a unified account of the correlations without sacrificing locality or altering the fundamental dynamics of the fields.

VII. CONCLUSION

A scalar–gauge field theory has been examined in which a single extended boundary configuration serves as the fundamental ontic element. The coupled structure (Σ, L)—consisting of a scalar-generated hypersurface and a gauge-fixed odd integer sector—possesses global identity conditions that cannot be decomposed into contributions associated with spatially disjoint regions. This non-separability distinguishes the boundary from ordinary field excitations and from all degrees of freedom that can be generated, modified, or annihilated by local perturbations.

Because (Σ, L) is defined only as a unified structure and is insensitive to bounded field deformations, it satisfies Bell's requirement for an entity that exists independently of measurement while lacking the separability properties presupposed by Bell's factorisation assumption. Joint outcome statistics for measurements performed in spacelike separated regions reflect the structural unity of the beable rather than any form of dynamical transmission. Local causality, microcausality, and the relativistic equations of motion remain intact.

The scalar–gauge model therefore provides a realist yet non-separable ontology in which the assumptions underlying Bell's theorem fail for structural reasons. The beable (Σ, L) furnishes a single, globally defined carrier of physical identity that grounds correlations without introducing nonlocal influences, hidden variables, or extensions of relativistic dynamics. This completes the ontology-first formulation of the model.

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