SP-1 — Rolling Criticality and Dual Universality Classes

Title

Rolling Criticality and Dual Universality Classes

Abstract

Experimental studies of a metastable crystal–fluid system operated under bounded cyclic perturbation have demonstrated repeatable work extraction, closed thermodynamic bookkeeping, and well-defined scaling behaviour reconstructed from measurements of pressure, temperature, volume, and mass. Earlier analysis successfully described these observations within a single universality class, providing a consistent effective account of the experimentally accessible observables.

The present work re-examines the experimental record at the level of response behaviour and susceptibility structure. A systematic empirical separation reveals the coexistence, within a single physical system and a single bounded cycle, of qualitatively distinct responses. These include smoothly propagating responses with finite susceptibility, abruptly saturating responses with effectively vanishing susceptibility, regimes of finite versus effectively gapless gradient-energy contribution, distinct response timescales, and bounded hysteric path dependence between expansion and compression stages. These features occur without phase change, material alteration, or breakdown of thermodynamic reconstruction.

It is shown that such behaviours cannot, in general, be reconciled within a single universality class. In particular, no smooth rescaling or continuous deformation can map responses exhibiting finite susceptibility and continuous scaling to responses exhibiting saturation, loss of susceptibility, or effectively gapped energetic behaviour without singularity. The minimal conclusion consistent with the experimental data is therefore that the system exhibits at least two inequivalent universality classes.

A descriptive condition termed *rolling criticality* is introduced to characterise how multiple universality classes may be accessed during cyclic operation without the occurrence of a global phase transition. This notion is purely structural and does not invoke renormalization-group flow, microscopic mechanisms, or field-theoretic interpretation.

The results are strictly classificatory. They establish the minimal universality structure required by the observed responses and defer questions of normalization, scale setting, and deeper theoretical interpretation to subsequent work.

1. Introduction

Recent experimental work has reported the behaviour of a metastable crystal–fluid system operated under controlled cyclic perturbations, with all thermodynamic quantities reconstructed from direct measurements of pressure, temperature, volume, and mass using standard equations of state [1]. Within that framework, the system was shown to support repeatable work extraction and well-defined scaling behaviour while remaining in a subcooled liquid phase throughout the cycle. On the basis of the experimentally accessible observables, the analysis in Ref. [1] successfully identified and applied a single universality class, within which the reconstructed responses were consistently described.

One empirical feature identified in Ref. [1] concerns the behaviour of the measured mass at different stages of operation. During the initial mixing process, a reduction in measured mass is observed, followed by an increase during subsequent transfer of the material into the piston expander. This increase persists over later stages of the cycle. The experiment itself does not determine how this behaviour should be interpreted, nor whether it reflects an effective inertial mass, a redistribution of internal interaction energies, or some other rearrangement within the overall mass–energy budget. What can be stated empirically is that the measured mass does not evolve monotonically and that its changes are correlated with distinct stages of the experimental protocol.

The earlier work also closed the thermodynamic bookkeeping at the level of internal energy, excess potentials, and work, identifying a constant energy Hamiltonian function under cyclic operation once wall-associated work terms are properly discounted [1]. In analysing the reconstructed scaling relations, it was further noted that several response functions admit natural parameterisations in terms of exponential and hyperbolic forms, and that certain quantities remain invariant under transformations resembling rapidity shifts rather than linear rescalings. These observations motivated the use of Lorentz-compatible kinematic language as a descriptive tool for organising the experimental data, without implying any underlying relativistic dynamics.

In addition to these kinematic regularities, the reconstructed cycle data exhibit further reproducible features that were not resolved within the original analysis. In particular, characteristic contraction of effective response times is observed in certain stages of the cycle when expressed in variables that render the scaling relations Lorentz-compatible. This behaviour can be described operationally as time contraction, although no interpretation in terms of time suppression, dynamical freezing, or microscopic causation is made at this stage.

A further feature concerns the behaviour of the reconstructed gradient-energy contribution. Across the cycle, this contribution separates into regimes in which it remains finite and regimes in which it effectively vanishes within experimental resolution. This separation is naturally described as a gap in the gradient-energy response, separating gapped from effectively gapless behaviour. The description is phenomenological and records a reproducible feature of the reconstructed energetics without assigning microscopic, geometric, or field-theoretic meaning.

Finally, the experimental cycle exhibits clear hysteric behaviour. Corresponding expansion and compression stages do not trace identical response paths, even when control variables are nominally reversed. This path dependence is repeatable across cycles and coexists with closed thermodynamic bookkeeping at the level of internal energy, excess potentials, and work. The presence of hysteresis therefore reflects a structural property of the response rather than irreversible degradation or loss.

Taken together, the successful application of a single universality class in Ref. [1], along with the appearance of Lorentz-compatible scaling, time contraction, gapped and gapless response regimes, and bounded hysteresis, provided a compact and internally consistent description of the experimentally accessible observables. However, the original analysis was intentionally restricted to that effective level and did not address whether these features exhaust the full scaling structure of the system, or whether additional response classes might be present but unresolved within the same framework.

The purpose of the present paper is to examine this classification question explicitly. The analysis proceeds at the level of response behaviour and scaling compatibility, without proposing a physical mechanism or microscopic explanation. The central question is whether a single universality class can, in general, accommodate the full set of experimentally observed responses when their susceptibility structure, path dependence, and coexistence within a single bounded cycle are taken into account.

The central result is a minimal classification statement: the observed response behaviours separate into at least two classes with incompatible scaling properties, such that no smooth rescaling or continuous deformation maps one class into the other. As a consequence, a faithful description of the system must recognise the presence of more than one universality class, even though a single class was sufficient for the original experimental reconstruction.

To account descriptively for how multiple response classes may be accessed during cyclic operation without the occurrence of a global phase transition, the notion of *rolling criticality* is introduced. This concept is used only to characterise the persistence of near-critical operation across the cycle while distinct response classes appear at different stages. It is not presented as a dynamical model and does not invoke renormalization-group flow or fixed-point machinery.

The scope of the present work is deliberately limited. No claims are made regarding microscopic mechanisms, geometric structure, gauge dynamics, or confinement phenomena. Although phenomenological similarities with dual superconductor behaviour have been noted previously—and such behaviour is widely discussed in the context of confinement mechanisms in non-Abelian gauge theories—these implications are not pursued here. Questions of normalization, deeper structural interpretation, and first-principles theoretical development are deferred to subsequent work. The sole objective of the present paper is to establish, on empirical and statistical-physics grounds, the minimal universality structure required to describe the observed responses consistently.

3. Universality Classes and Scaling Incompatibility

In statistical physics, a universality class is defined by the existence of a common scaling description that relates response functions across control parameters, histories, and observables. Systems belonging to a single universality class admit continuous tuning of susceptibility, smooth interpolation between response regimes, and a shared normalization structure, such that differences in behaviour can be absorbed into rescalings of variables rather than requiring distinct descriptions.

Applied to the present system, a single-universality description would therefore require that all observed response behaviours—suppressed, propagating, hysteric, gapped, and effectively gapless—be mutually compatible under such continuous rescaling. In particular, it would require that apparent loss of susceptibility, disappearance of gradient-energy contributions, contraction of response times, and path dependence under reversal of control variables can all be understood as limiting cases of the same underlying scaling law.

The empirical separations documented in Section 2 place strong constraints on this possibility. Suppressed response segments are characterised by abrupt saturation and effective insensitivity to further perturbation, rather than by continuously decreasing susceptibility. In these regimes, entire response channels effectively vanish within experimental resolution. By contrast, propagating response segments display smooth variation, well-defined susceptibility, and compatibility with continuous scaling forms. The transition between these behaviours occurs sharply on the timescale of the cycle and does not exhibit the gradual crossover expected if both belonged to a single scaling family.

A similar incompatibility arises in the behaviour of the reconstructed gradient-energy contribution. The separation between regimes in which this contribution is finite and regimes in which it effectively vanishes is not accompanied by continuous softening or divergence characteristic of crossover within a single universality class. Instead, the data exhibit a clear distinction between gapped and effectively gapless behaviour, with no intermediate regime that would allow one to be obtained from the other by smooth deformation.

The presence of bounded but persistent hysteresis further strengthens this incompatibility. Within a single-universality framework, hysteric effects can often be absorbed into renormalized parameters or treated as perturbative corrections around an underlying reversible scaling law. In the present system, however, path dependence is not confined to small deviations but is structurally tied to the distinction between suppressed and propagating response segments. The system follows different response paths under reversal of control variables even when macroscopic state variables coincide, indicating that history dependence is not a marginal effect but an intrinsic feature of the response structure.

Time contraction provides an additional and independent indication of scaling incompatibility. Response segments associated with suppressed behaviour exhibit markedly different effective timescales from those associated with propagating behaviour when expressed in variables that render the scaling relations Lorentz-compatible. These timescale differences are not continuously connected across the cycle and cannot be removed by reparameterization without eliminating the suppressed regimes altogether.

Taken together, these features rule out a single-universality description in which all observed responses are related by smooth rescaling or continuous interpolation. The incompatibility does not arise from numerical thresholds, experimental noise, or insufficient proximity to criticality, but from qualitative differences in susceptibility structure, response continuity, and history dependence. No single set of scaling relations can simultaneously accommodate regimes of vanishing response, finite propagating response, gapped and effectively gapless energetic behaviour, and bounded hysteresis within one continuous framework.

The minimal conclusion consistent with the empirical record is therefore that the system exhibits more than one universality class. These classes are not introduced here as physical entities or distinct phases, but as inequivalent sets of scaling behaviour required to describe the observed responses. One class captures the continuously scaling, propagating responses with well-defined susceptibility and Lorentz-compatible kinematic structure. The other captures the suppressed, saturating, and effectively gapped responses associated with abrupt loss of susceptibility, contracted response times, and intrinsic hysteresis.

This conclusion is purely classificatory. It does not assign microscopic mechanisms, spatial separation, or underlying degrees of freedom to the different universality classes. It establishes only that no single universality class can faithfully represent the full set of observed response behaviours. The next section introduces a descriptive framework for understanding how multiple universality classes may be accessed within a single bounded cycle of operation without the occurrence of a global phase transition.

4. Minimal Dual Classification

Having established that a single universality class cannot accommodate the full set of observed response behaviours, the remaining task is to formulate the minimal classification consistent with the empirical record. The aim of this section is not to propose a detailed model or hierarchy, but to introduce the smallest number of response classes required to represent the demonstrated scaling incompatibilities.

The empirical separations identified in the preceding sections indicate that the response behaviours fall naturally into two groups distinguished by their susceptibility structure, continuity of response, and dependence on perturbation history. Introducing more than two classes is not warranted at this stage, as the available data do not compel a finer subdivision. Conversely, collapsing all observed behaviours into a single class has already been shown to be untenable. The appropriate description is therefore a minimal dual classification.

The first response class is characterised by continuously varying behaviour with well-defined susceptibility. In this class, reconstructed response quantities change smoothly with applied perturbation and admit consistent scaling parameterisations across repeated cycles. Responses propagate without abrupt saturation, and effective response times remain extended relative to the cycle timescale. This class encompasses those stages of the cycle in which the system exhibits Lorentz-compatible kinematic structure and smooth interpolation between neighbouring operating points.

The second response class is characterised by abrupt saturation and effective loss of susceptibility. In this class, continued perturbation produces little or no further change in the relevant response variable within experimental resolution. Entire response channels may effectively vanish, and reconstructed gradient-energy contributions may drop to zero. Effective response times are contracted relative to those observed in the continuously scaling class, and behaviour is strongly dependent on perturbation history, giving rise to intrinsic hysteresis.

These two classes are defined entirely by their response characteristics. No physical interpretation is attached to them here, and no claim is made that they correspond to distinct phases, spatial regions, or microscopic degrees of freedom. They are not introduced as ontological categories, but as a minimal structural classification required to represent the observed data faithfully.

Importantly, the classification does not imply that the system alternates between two disconnected states. Both response classes are accessed within a single physical system and within a single bounded cycle of operation. The distinction lies not in the identity of the system, but in the scaling behaviour it exhibits at different stages of the cycle. The same control variables and reconstruction framework apply throughout, yet the response structure changes qualitatively.

The dual classification introduced here is therefore purely operational. It captures the irreducible difference between response behaviours that admit continuous scaling and those that exhibit suppressed or excluded response. It does not explain why these behaviours arise, nor does it assign causal mechanisms. Its sole purpose is to provide a clear and minimal framework within which the incompatibility demonstrated in Section 3 can be expressed.

The following section addresses how a system exhibiting such a dual classification can nonetheless operate continuously and repeatably under cyclic perturbation, without undergoing a global phase transition.

5. Rolling Criticality as Structural Selection

The dual classification introduced in the preceding section raises a natural descriptive question: how can a single physical system, operating under bounded and repeatable cyclic perturbation, access two inequivalent universality classes without undergoing a global phase transition? The purpose of this section is to address this question at a structural level, without invoking microscopic mechanisms, dynamical evolution equations, or renormalization-group concepts.

The experimental cycle does not exhibit signatures of a conventional phase transition. The system remains in a subcooled liquid state throughout, thermodynamic reconstruction remains valid, and macroscopic state variables return to their initial values at the completion of each cycle. Nonetheless, the response behaviour changes qualitatively between stages of the cycle, as evidenced by the appearance and disappearance of susceptibility, the alternation between propagating and suppressed responses, the presence or absence of gradient-energy contributions, and the emergence of hysteric path dependence.

These observations motivate the introduction of *rolling criticality* as a descriptive condition. In this context, rolling criticality refers to the persistence of near-critical operation across the cycle, such that the system repeatedly approaches, but does not cross, a critical boundary in response space. As external perturbations are applied and reversed, different response classes become accessible while the system as a whole remains bounded and stable. The critical boundary is not traversed in the sense of a phase transition; rather, it is approached from different sides at different stages of the cycle.

Under rolling criticality, the activation of a given response class is determined structurally by the position of the system relative to this critical boundary at a particular stage of the cycle. When the system operates on one side of the boundary, responses with well-defined susceptibility and continuous scaling are expressed. When it operates on the other side, susceptibility is suppressed, response channels saturate, and effectively gapped behaviour emerges. The system therefore selects between response classes as it rolls along the critical boundary, without collapsing into a single dominant regime.

This notion of selection is structural rather than causal. Rolling criticality does not describe a mechanism by which the system is driven into one class or the other, nor does it specify how the critical boundary itself arises. It serves only to characterise the observed coexistence of multiple response classes within a single cycle and to clarify why this coexistence does not contradict the absence of a global phase transition.

Importantly, rolling criticality does not imply the existence of multiple phases, spatial regions, or distinct degrees of freedom. The classification remains entirely at the level of response behaviour. The same system, described by the same reconstructed variables and operating under the same experimental protocol, exhibits different scaling structures at different stages of the cycle because its proximity to a critical boundary varies continuously in time.

In this sense, rolling criticality provides a minimal structural account of how dual universality classes can be realised operationally within a single bounded experiment. It neither explains the origin of the critical boundary nor resolves the deeper source of the incompatibility between response classes. Those questions are explicitly deferred. At the present level, rolling criticality functions as a descriptive bridge between the empirical separation of response behaviours and the minimal dual classification required to represent them consistently.

6. Explicit Non-Claims

The results presented in this paper are intentionally limited in scope. To avoid misinterpretation, it is important to state explicitly what is *not* being claimed.

No assertion is made regarding the microscopic origin of the observed response behaviours. The analysis does not invoke specific degrees of freedom, interaction mechanisms, or internal structures responsible for the suppressed or propagating responses. The classification presented here is entirely agnostic with respect to underlying physical constituents.

No claim is made that the dual universality classes identified here correspond to distinct thermodynamic phases, spatial regions, or separable subsystems. The system remains a single, coherent physical entity throughout the experimental cycle, and the classification refers solely to response behaviour under perturbation, not to material composition, phase separation, or spatial partitioning.

No interpretation is offered in terms of gauge structure, topological order, confinement mechanisms, or related field-theoretic concepts. Although phenomenological similarities with such frameworks may exist, they are not required to establish the present classification and are deliberately excluded from the analysis.

No renormalization-group flow, fixed-point structure, or dynamical critical theory is assumed. The notion of rolling criticality is introduced solely as a descriptive condition characterising how different response classes can be accessed during cyclic operation without a global phase transition. It does not imply a specific critical theory, scaling hierarchy, or dynamical instability.

No claims are made regarding gravitational analogies, relativistic dynamics, or the fundamental nature of time. Lorentz-compatible kinematic structure and time contraction are treated as observed features of the reconstructed scaling relations, not as evidence of spacetime symmetry, temporal suppression, or causal modification.

Finally, no quantitative normalization or scale-setting is attempted in this work. While the experimental system exhibits well-defined energy, mass, and timescale values, these are not used here to anchor universality classes or to derive absolute constants. Questions of normalization, amplitude setting, and the origin of specific scale values are explicitly deferred to subsequent analysis.

Taken together, these non-claims delineate the present work as a strictly empirical and structural classification of response behaviour. The results neither require nor preclude deeper theoretical interpretation. Their purpose is to establish, in the weakest form consistent with the data, the minimal universality structure demanded by the observed responses.

7. Transition to Normalization

The analysis presented in this paper establishes a strictly empirical and structural result: the experimentally observed system exhibits at least two inequivalent universality classes distinguished by incompatible scaling behaviour and susceptibility structure. This conclusion follows directly from the coexistence, within a single bounded cycle, of suppressed and propagating responses, gapped and effectively gapless energetic behaviour, distinct response timescales, and bounded hysteresis. No smooth rescaling or continuous deformation suffices to map one class into the other.

At this stage, the classification is deliberately incomplete. While the existence of multiple universality classes has been established, no attempt has been made to determine how physical scales are set within each class, how amplitudes are normalized, or how quantitative values arise from the experimental data without arbitrary insertion. These questions are orthogonal to the classification result and cannot be resolved at the same level of analysis without presupposing the outcome.

The present work therefore closes at the point where classification ends and normalization begins. Having established that the system admits at least two inequivalent universality classes, the next task is to determine how normalization constants arise within each class without arbitrary scale insertion.

Appendix A

Formal Statement of Scaling Incompatibility

This appendix provides a minimal formalisation of the scaling incompatibility underlying the classification result presented in the main text. It introduces no additional assumptions and does not extend the scope of the analysis.

Consider a generic response observable Rmeasured as a function of a control parameter λduring a given stage of the experimental cycle. The response may also depend implicitly on the direction of perturbation and on history, but these dependences are suppressed here for clarity unless explicitly stated.

Define the susceptibility associated with this response as

$$\chi(\lambda) \equiv \frac{\partial R}{\partial \lambda}.$$

In the standard statistical-physics sense, two responses $R_1(\lambda)$and $R_2(\lambda)$are said to belong to the same universality class if there exists a smooth, invertible reparameterisation $f$such that

$$R_1(\lambda) = f(R_2(\lambda)),$$

with f continuously differentiable over the domain of interest. Under such a mapping, qualitative features of the response—such as continuity, monotonicity, and the existence of finite susceptibility—are preserved up to rescaling.

Now consider two empirically observed response behaviours:

Propagating response.
There exists an open interval $I \subset \mathbb{R}$ such that

$$\chi_{\text{prop}}(\lambda) \neq 0 \text{ for all } \lambda \in I,$$

and $\chi_{\text{prop}}(\lambda)$ varies continuously with λ. This response admits smooth interpolation and continuous scaling.

Suppressed response.
There exists an open interval $J \subset \mathbb{R}$ such that

$$\chi_{\text{sup}}(\lambda) = 0 \text{ for all } \lambda \in J,$$

within experimental resolution. Over this interval, the response saturates: changes in λ do not induce changes in R.

Assume, for contradiction, that these two responses belong to the same universality class. Then there must exist a smooth, invertible mapping f such that

$$R_{\text{sup}}(\lambda) = f\big(R_{\text{prop}}(\lambda)\big) \text{ for } \lambda \in I \cap J.$$

Differentiating with respect to λ yields

$$\chi_{\text{sup}}(\lambda) = f'\big(R_{\text{prop}}(\lambda)\big)\,\chi_{\text{prop}}(\lambda).$$

On $I \cap J$, the left-hand side vanishes identically, while $\chi_{\text{prop}}(\lambda) \neq 0$. Therefore,

$$f'\big(R_{\text{prop}}(\lambda)\big) = 0 \text{ for all } \lambda \in I \cap J.$$

This implies that f is constant over a finite interval of its argument, which contradicts the requirement that f be invertible and continuously differentiable. The only alternatives are that f is singular or non-invertible, in which case the two responses cannot be related by universality-preserving rescaling.

An analogous argument applies to the reconstructed gradient-energy contribution. Let $G(\lambda)$denote this contribution, and suppose that

$$G_{\mathrm{prop}}(\lambda) > 0 \text{on an open interval,}$$

while

$$G_{\mathrm{sup}}(\lambda) = 0 \text{on an open interval.}$$

No smooth reparameterisation can map a finite-valued function to one that vanishes identically over a finite domain without singular behaviour.

Finally, consider hysteric response, characterised by

$$R(\lambda, \uparrow) \neq R(\lambda, \downarrow),$$

where ↑and ↓denote opposite directions of perturbation. If this path dependence were perturbative, it could be absorbed into a renormalisation of parameters within a single universality class. However, when hysteresis coincides with vanishing susceptibility or saturation of response, it reflects a structural difference in scaling behaviour rather than a marginal correction.

These formal considerations establish that response behaviours exhibiting finite susceptibility and continuous scaling cannot be related by smooth rescaling to behaviours exhibiting vanishing susceptibility, saturation, or effectively gapped energetic contributions. The incompatibility is structural and does not depend on numerical thresholds, proximity to criticality, or choice of parametrisation.

Accordingly, the coexistence of such behaviours within the experimental system necessitates the introduction of more than one universality class, as stated in the main text.

Appendix B

Hysteresis and Constraints on Universality

This appendix formalises the role of hysteric response in constraining universality-class descriptions. The purpose is to distinguish hysteresis that is compatible with single-class scaling from hysteresis that is structurally incompatible with such a description.

Let $R(\lambda)$denote a reconstructed response observable as a function of a control parameter λ. Define two response branches corresponding to opposite directions of perturbation:

$$R^{+}(\lambda)(\text{increasing } \lambda), R^{-}(\lambda)(\text{decreasing } \lambda).$$

Hysteresis is present if

$$R^{+}(\lambda) \neq R^{-}(\lambda)$$

over some interval of λ. This condition alone does not preclude a single universality-class description. In many systems, hysteresis arises from finite response times, lag effects, or dissipative delays and can be treated as a perturbation around an underlying reversible scaling law.

For hysteresis to be compatible with a single universality class, two conditions must hold.

Finite susceptibility on both branches.
The susceptibilities

$$\chi^{\pm}(\lambda) \equiv \frac{\partial R^{\pm}}{\partial \lambda}$$

must remain finite and non-zero over the interval of hysteresis.

Continuous branch connection.
There must exist a smooth mapping f such that

$$R^{+}(\lambda) = f(R^{-}(\lambda)),$$

with f invertible and continuously differentiable. Under this condition, hysteresis corresponds to a deformation within a single scaling family rather than to a distinct scaling structure.

The experimentally observed hysteresis documented in the main text violates these conditions. Specifically, hysteric path dependence coincides with regimes in which susceptibility vanishes or response saturates. In such regimes,

$$\chi^{\pm}(\lambda) = 0$$

over finite intervals, implying that the response is insensitive to changes in λ on at least one branch.

Under these circumstances, no smooth, invertible mapping can relate the two branches without singular behaviour. Any attempt to construct such a mapping would require collapsing a finite interval of one branch onto a point or plateau of the other, violating the requirements of universality-preserving rescaling.

Furthermore, the hysteresis observed in the experimental cycle is bounded and repeatable: the system returns to the same macroscopic state at the completion of each cycle, and thermodynamic bookkeeping remains closed. This excludes explanations based on cumulative dissipation, drift, or degradation. The hysteresis therefore reflects a structural distinction between response regimes rather than a transient or marginal effect.

Formally, the coexistence of hysteresis with vanishing susceptibility implies that the response cannot be represented as a single-valued function of $\lambda$with small corrections. Instead, the response space decomposes into inequivalent branches distinguished by their susceptibility structure. This decomposition is incompatible with a single universality class.

Accordingly, hysteresis in the present system does not merely decorate an otherwise uniform scaling law. When combined with suppressed response and effectively gapped energetic behaviour, it provides an independent and complementary obstruction to single-universality descriptions. This conclusion supports, but does not extend beyond, the minimal dual classification established in the main text.

Appendix C

Rolling Criticality as a Structural Condition

This appendix formalises the notion of *rolling criticality* introduced in the main text by characterising it as a structural condition on response behaviour. The purpose is to clarify how multiple universality classes may be accessed within a single bounded experiment without invoking dynamical evolution equations, renormalization-group flow, or phase-transition machinery. No additional mechanisms or assumptions are introduced.

Consider a system characterised by a set of reconstructed response observables $\{R_i\}$, each depending on a control parameter λ, and evolving under a bounded cyclic protocol. Let

$$\chi_i(\lambda) \equiv \frac{\partial R_i}{\partial \lambda}$$

denote the corresponding susceptibilities.

In conventional treatments of critical phenomena, criticality is associated with either
(i) approach to a fixed critical point under variation of a control parameter, or
(ii) traversal of a critical surface separating distinct thermodynamic phases.
Both cases involve singular behaviour in susceptibilities or correlation lengths and are typically described using renormalization-group flow or fixed-point structure.

The experimental system considered here exhibits neither of these features. Instead, the following empirical properties are observed:

- the system remains within a single thermodynamic phase throughout the cycle;
- susceptibilities remain finite in propagating response regimes and vanish in suppressed regimes, without divergence;

- response behaviour alternates between inequivalent scaling structures as the cycle progresses;
- macroscopic state variables return to their initial values at the completion of each cycle.

Rolling criticality is introduced to describe precisely this situation. Formally, it refers to the existence of a critical boundary $\mathcal{B}$in response space such that:

1. the system remains within a neighbourhood of $\mathcal{B}$throughout the cycle;
2. the cycle trajectory approaches $\mathcal{B}$from different sides at different stages of the cycle;
3. the susceptibility structure changes discontinuously across $\mathcal{B}$, while the system does not cross into a distinct thermodynamic phase.

This structure may be expressed schematically by partitioning the response space into regions

$$\mathcal{R}_{\text{prop}} \text{and} \mathcal{R}_{\text{sup}},$$

corresponding to propagating and suppressed response classes, respectively, with

$$\mathcal{R}_{\text{prop}} \cap \mathcal{R}_{\text{sup}} = \emptyset,$$

and a boundary

$$\partial\mathcal{R} = \mathcal{B}$$

separating them. During cyclic operation, the system trajectory intersects neighbourhoods of both regions while remaining within the admissible response domain.

Crucially, rolling criticality does not imply tuning through a critical point, nor does it require the existence of diverging length or time scales. The alternation between response classes reflects proximity to the boundary $\mathcal{B}$, not passage through it. As a result, no renormalization-group description is implied or required.

The term *rolling* emphasises the temporal aspect of this condition: as the system is driven cyclically, it continuously adjusts its position relative to the critical boundary, allowing different response classes to be expressed at different stages. The critical boundary itself is treated as a structural feature of the response space, not as an emergent object derived from microscopic dynamics.

This formulation clarifies that rolling criticality serves only as a descriptive framework for organising the observed coexistence of multiple universality classes within a single bounded experiment. It neither explains the origin of the critical boundary nor prescribes how it should be computed. Those questions lie outside the scope of the present work and are explicitly deferred to subsequent analysis.

References

[1] M. Gibbons, *Superconducting Phase Transition in a Metastable Crystal–Fluid System*, **Journal of Physics D: Applied Physics 56**, 054001 (2023).